B.Sc.
Part I (Hons). Paper 1st

Q. Inverse of $\begin{bmatrix} 3 & 5 & 7 \\ 2 & 3 & 0 \\ 5 & 9 & 2 \end{bmatrix}$

$\Rightarrow$ Let $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & 3 & 0 \\ 5 & 9 & 2 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} 3 & 5 & 7 \\ 2 & 3 & 0 \\ 5 & 9 & 2 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 5 & 7 \\ 9 & 2 \end{vmatrix} + 3 \begin{vmatrix} 3 & 7 \\ 5 & 2 \end{vmatrix}$$

$$= -2(10 - 63) + 3(6 - 35)$$

$$= (-2) \times (-53) + 3 \times (-29)$$

$$= 106 - 87 = 19$$

$\therefore |A| \neq 0 \Rightarrow A^{-1}$ is possible.

Now, cofactors of 1st row of $A =$

$$\begin{vmatrix} 3 & 0 \\ 9 & 2 \end{vmatrix}, \; - \begin{vmatrix} 2 & 0 \\ 5 & 2 \end{vmatrix}, \; \begin{vmatrix} 2 & 3 \\ 5 & 9 \end{vmatrix}$$

$$= 6, -4, 3$$

cofactors of 2nd row

$$= -\begin{vmatrix} 5 & 7 \\ 9 & 2 \end{vmatrix}, \begin{vmatrix} 3 & 7 \\ 5 & 2 \end{vmatrix}, -\begin{vmatrix} 3 & 5 \\ 5 & 9 \end{vmatrix}$$

$= 53, \quad -29, \quad -2$

cofactors of elements of 3rd row

$$= \begin{vmatrix} 5 & 7 \\ 3 & 0 \end{vmatrix}, -\begin{vmatrix} 3 & 2 \\ 7 & 0 \end{vmatrix}, \begin{vmatrix} 3 & 5 \\ 2 & 3 \end{vmatrix}$$

$= -21, \quad 14, \quad -1.$

$$\therefore \text{Adjoint of } A = \begin{bmatrix} 6 & 53 & -21 \\ -4 & -29 & 14 \\ 3 & -2 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{Adjoint of } A}{|A|}$$

$$= \frac{1}{19}\begin{bmatrix} 6 & 53 & -21 \\ -4 & -29 & 14 \\ 3 & -2 & -1 \end{bmatrix}$$